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THE SHADOW WAGE RATE AND THE NUMBERS EFFECT*

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I. INTRODUCTION

The Shadow Wage Rate (SWR), like any shadow price, is a function of the social objectives set for the particular country. Brent [1984 and 1986] has argued that one should consider the number of uncompensated losers from a public investment as a third social objective (together with the usual efficiency and distributional objectives). This paper explores the implications for the SWR of Squire and van der Tak [1976] (S & T) of having this third objective. It uses a simple optimal control model of the type used by Lal and Squire [1980] to derive the SWR. The general implications for S & T's methods of including the numbers effect have been explored in Brent [1990]. It will be via a concern for employment that the numbers effect will enter the SWR. Employment has long been an important consideration for policy makers. Now analysts can have a vehicle for incorporating this consideration.

We begin the analysis in Section II by reconstructing the three component S & T SWR formula. An important implication for estimation is then examined. In the next section, this framework is extended to include the numbers effect. The numbers effect makes an allowance for the encouragement of employment as a social objective in its own right. In Section IV attention is given to migration and its impact on the SWR. Here it is through the converse of employment, i.e., unemployment, that the numbers effect is having its influence on the SWR. This consideration forms the new fourth component in the S & T SWR formula, the “Social Cost of Unemployment”. Some idea of the size of the numbers effect on the SWR is given in Section V, prior to presenting the conclusions.

II. THE S & T SHADOW WAGE RATE

The underlying process is assumed to be one where unskilled labor is transferring from the rural area to the urban area in order to obtain employment on the public project. Both the urban wage rate \( u \) and the rural wage rate \( m \) are taken to be fixed. It is fundamental to the analysis that \( u > m \), for this is what attracts the labor to the urban area. In the L & M model \( u \) and \( m \) are fixed by institutional factors. More generally, \( u > m \) can be due to demand (productivity) factors, due to greater labor endowments and educational training in the urban areas, or due to supply factors, such as differences in differential employment or activity rates in urban and rural areas. Note that skilled labor can be considered as a special case
of the subsequent analysis by setting \( w = m \). The value of public income is the
numeraire. The value of the numeraire relative to private consumption is given by \( v \).

There are three cost components in the S & T SWR, representing the two social
objectives, efficiency and distribution. The first is forgone output, measured by \( m \). It
is expressed in term of the numeraire by the output conversion factor \( \alpha \). The second
is extra consumption induced by the employment, given by the difference in wages
between the two areas \( w - m \). This has a net cost reflected by the difference between
the consumption conversion factor \( \beta \) and the distributional weight attached to the
consumption relative to the value of the numeraire \( d/v \). The final cost is the social
evaluation \( \phi \) attached to the workers evaluation \( e \) of the disutility of working for the
extra wage \( w - m \). Work effort is treated like any other good. The relative distribution
weight \( d/v \) is applied to it. The SWR is the sum of the three cost components.\(^1\)

\[
SWR = m\alpha + (w - m)(\beta - d/v) + (w - m)\phi e/d/v
\]

S & T do not provide a formal derivation of their SWR. However, their formula is
based on the SWR by Little and Mirrlees [1974] (L & M), which has been derived in
a simple fashion by Lal and Squire. So we shall convert S & T’s SWR formula into
L & M’s version in order that we can use Lal and Squire’s derivation. The easiest
way of making the conversion is to ignore the third component of the S & T formula
(assume \( \phi = 0 \)) and set \( \alpha = \beta = d = 1 \). After rearrangement, this leads to
the L & M SWR.\(^2\)

\[
SWR = w - (w - m)(1/v)
\]

Squire and Lal obtain eq. (2) from a dynamic optimisation problem. The objective is
social welfare \( V \), which is a concave function dependent on changes in employment-
generated consumption \( (w - m)L \), where \( L \) is urban employment:

\[
V = \int_0^\infty V((w - m)L)dt
\]

The constraint is that investment \( \dot{K} \) is that part of urban output that is not consumed.
With urban output a function of \( K \) (urban capital) and \( L \), and urban consumption
given by \( wL \), the costate equation is

\[
\dot{K} = f(K, L) - wL
\]

The problem then is to maximise \( V \), as in eq. (3), subject to the constraint, given by
eq (4). All variables except \( m \) and \( w \) are functions of time. \( L \) is the control variable.
and $K$ is the state variable. The Hamiltonian, with $\lambda$ as the utility price of investment, is

$$
H = V[(w - m)L] + \lambda[f(K, L) - wL]
$$

Necessary conditions for an optimum path are:

$$
H_L = (w - m)V' + \lambda f_L - \lambda w = 0
$$

$$
H_K = \lambda f_K = -\lambda
$$

The SWR can be defined as the social value of the marginal product of labor. Since the problem represented by (5) is one of finding a utility maximisation path, $\lambda f_L$ is the SWR in utility terms. The S & T SWR, with investment as the numeraire, divides the utility SWR by $\lambda$. In which case the SWR is simply $f_L$. From eq. (6) we obtain

$$
SWR = m + (w - m)(1 - V'/\lambda)
$$

Eq. (8) is equal to L & M’s (and S & T’s) eq. (2), by grouping terms in $m$, and defining $\lambda/V' = v$. In line with S & T’s full formulation, the first term in eq. (8) is the foregone output effect, and the second term is the social cost of extra consumption.

The Squire and Lal model does have an important empirical implication that follows from the way that we have just defined $v$. From eq. (6),

$$
v = \frac{w - m}{w - f_L}
$$

$v$ does play a very significant role in the whole S & T methodology. It is interesting that one can obtain from the SWR framework an independent estimate of $v$ that is based entirely on objectively measurable parameters, i.e., the sector wage gap relative to the urban wage surplus.

III. THE NUMBERS EFFECT AND EMPLOYMENT

The previous section has shown that the SWR is dependent on both the form of the social objective function and the nature of the constraints. Sah and Stiglitz [1985] examine alternative specifications of the constraints. In this paper we reassess the welfare function. The purpose of this section is to explain the important underlying concepts, and to illustrate how S & T’s model is to be extended. In Section IV the main model is presented.

The original motivation for including the numbers effect as a third social objective in public expenditure analysis came from an empirical study of UK government railway closure decisions (Brent [1976]). The aim of the study was to estimate the implicit distributional weights behind past government closure decisions. But, when one came across the data in the files that the decision-maker actually used to make the decisions, the numbers affected was allegedly of prime concern. The results confirmed this. The number of people who were to be left with no viable public alternative means of transport was a significant determinant of whether an unremunerative railway line was to be closed or left open.

In addition to this empirical result, there is a theoretical reason, given in Brent [1986], for wanting to include the numbers effect as a third factor. It has been shown (in a poverty context) that no social welfare function based solely on efficiency and linear distribution weights can be sensitive to the number of losers (the number of persons in poverty). The numbers effect is an additional factor to the other two. An important implication of this theorem is that one cannot say that the numbers effect is just a proxy for the efficiency objective.

If social welfare is to be determined by more than efficiency and distribution, it is important to know how this fits in with the welfare economic base behind cost-benefit analysis and project appraisal. The foundation stone behind these types of analyses is the idea of a Kaldor-Hicks compensation test. A project makes society better off if there are sufficient gains such that the losers can be compensated, and there is still some positive amount left over. However, it does ignore the possibility that, for real world projects, it is impossible administratively to arrange for compensation for all. Almost always, there will be losers. It is true it has been recognised in the literature that the existence of losers is a problem if they are a low income group; the response then being to apply distribution weights to the net-benefits of the gainers and losers. But, the literature has overlooked the existence of losers when income distribution is not affected. It would seem consistent with the underlying theory to record the number of uncompensated losers, and use this information when making public expenditure decisions. The use of the number of uncompensated losers means that the employment objective will always be actually considered; it is not going to be an hypothetical consideration, like the Kaldor-Hicks test that applied to the distributional objective.

It is important to consider another way of interpreting the role of the numbers effect in welfare economic terms. The aggregate size of the net-benefits (economic efficiency) has long been recognised as an inadequate index of the complete social effects of a project. Many have focused on the problem posed by the existence of large gains for the few at the expense of small losses for a much larger number. The standard solution has been to appeal to distribution weights. Even though this has merit in principle, there still has been some unease in the literature with the making of interpersonal comparisons. Given this, recording the number of losers enables one to go beyond economic efficiency, to consider also one aspect of its distribution, yet
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The shadow wage rate is the money wage that is needed to replace the real wages of being employed. It is measured as the money wage that must be paid to an individual to make them indifferent between working and not working. When we consider the effects of changes in the labor market, we need to consider the shadow wage rate. The shadow wage rate is useful in determining the impact of changes in the labor market on employment and wages.

The shadow wage rate is calculated as follows:

\[ W^* = \frac{V}{m} \]

where 

- \( W^* \) is the shadow wage rate
- \( V \) is the marginal utility of income
- \( m \) is the marginal propensity to save

In the context of labor economics, the shadow wage rate is used to determine the impact of changes in the labor market on employment and wages. The shadow wage rate is used to determine the labor force participation and labor supply decisions of individuals.

The shadow wage rate is also used to determine the effects of changes in the labor market on the distribution of income. The shadow wage rate is used to determine the impact of changes in the labor market on the distribution of income and the distribution of wealth.

In conclusion, the shadow wage rate is a useful tool in determining the impact of changes in the labor market on employment and wages. It is used to determine the labor force participation and labor supply decisions of individuals and the distribution of income and wealth.
The empirical implication contained in eq. (11), that now follows from the SWR optimisation framework is

\[ a = \frac{w(w - f_L) - (w - m)}{w} \]  

A value for \( a \) is necessary to make most development plans operational. Provided that one obtains an estimate for \( v \) using standard S & T methods, the trade-off between employment and consumption can be inferred from measurable parameters. This time it is the difference between the (weighted) wage surplus and the wage gap, relative to the urban wage, that gives us important value information.

The problem with the current formulation is that it assumes that society is only concerned with \( L \), employment in the urban sector. A wider perspective is necessary, and this takes place in the next section.

IV. THE NUMBERS EFFECT AND UNEMPLOYMENT

Once one considers employment in the economy as a whole, and how employment in one sector may come at the expense of employment in the other, it is appropriate to focus on the converse of employment, i.e., unemployment. Central to the idea of urban job creation and its effect on overall unemployment in LDC's is the rural-urban migration process. For our purposes, the precise migration mechanism is not important. What is of interest is the end result in terms of unemployment that follows from the migration.

Before introducing the link between migration and unemployment into the analysis, it is first necessary to mention how the other components of the SWR are affected by migration.

Following Lal [1973, eq. (3)], migration \( M \) can be inserted into the SWR by replacing \( m \) by \( M.m \) (there are \( M \) people who forego the rural marginal product \( m \)). After the substitution of this value into the simple S & T version, eq. (8), the SWR becomes

\[ SWR = M.m + (w - M.m)(1 - V'/\lambda) \]  

As Lal points out, if one assumes that the migration process follows the Todaro [1969] model, then this means that \( M = w/m.8 \) Eq. (14) then produces the Harberger [1971] result. The SWR is given by the urban wage rate \( w \). Clearly, this result does not follow if one considers non-unity values for the parameters \( \alpha, \beta \) and \( d \). The SWR will also differ from \( w \) when we now incorporate unemployment into the migration process.

We will say that migration is "efficient" when there is one person transferred out of the rural area for each urban job created. In these circumstances (and assuming, as with S & T, no unemployment in the rural sector) migration may lead to no net gain in terms of the overall employment objective.9 However, typically more than one person is attracted by the possibility of an urban job. In which case, urban job creation actually leads to an increase in unemployment. This follows from the basic definition of (urban) unemployment

\[ U = N - L \]

where \( N \) is the size of the urban labour force, and \( U \) is the number unemployed. The effect on unemployment from urban job creation is given by

\[ dU/dL = dN/dL - 1 = M - 1 \]

where \( dN/dL \) signifies the size of rural-urban migration (per job created on the project), represented by \( M \). Only if migration is efficient will \( M = 1 \). In general then, one can expect the unemployment derivative in eq. (16) to be positive. \(-(M - 1)\) measures the extent to which employment is \( not \) being furthered.

In the more general circumstances where rural-urban migration is a factor, it is the unemployment index \( -(M - 1) \) that is the better gauge of the employment effects of the public project. To form the employment objective, the rural wage \( m \) that is being foregone should be used as the weight to indicate the social value of the lost job.10 Using a weighted unemployment specification for the employment objective, we can rewrite the objective function as

\[ V = \int_0^\infty V[(w - m) L, -m(M - 1)] dt \quad V_1 > 0, V_2 < 0 \]

The SWR now takes the form:11

\[ SWR = w - (w - m)(1/v) + am(M - 1)(1/v) \]

where \( am(M - 1)(1/v) \) is the "social cost of unemployment". This raises the SWR.

A comparison of eq. (18) with S & T's eq. (2) shows that the numbers effect enters additively into the SWR. We can therefore return to the full S & T formula and insert it as a fourth component to form

\[ SWR = m\alpha + (w - m)(\beta - d/v) + (w - m)\beta d/v + am(M - 1)(d/v) \]

Note two aspects of eq. (19). Firstly, the distributional weight \( d \) is not necessarily equal to 1. Hence in eq. (19), the social cost of unemployment is weighted by \( d/v \) rather than just by \( 1/v \). In this way it is weighted like the cost of leisure term, i.e., treated like any other good. Secondly, it was written without the migration effect for the non-numbers effect components. Following eq. (14), it is a simple matter to include
VI. CONCLUSIONS

Much has been written on whether the SWF can be taken to be less than the urban wage rate (see, for example, Harberger, 1981; Pindyck, 1985). The argument that large-scale migration is not more important because the urban wage rate is lower may also be used here. However, including the numbers effect in the SWF may provide another dimension to this debate. As was shown in Section III, if one is only concerned about employment in the urban sector, the SWF may be lower with the numbers effect included, but in the more general case, where one is concerned about the overall level of employment, that may be higher. For this reason, the SWF will be lower and the social welfare benefits of employment creation may not be as large as one would think.
To confirm this statement, just insert \( M = 0 \) into eq. (19) to see that the sign of the fourth component becomes negative.

An anonymous referee has emphasized to me the existence of differential employment or activity rates in urban and rural areas. The implication of this differential activity rate is that the probability that it would be possible to attract new labor in an urban area that is not yet employed is lower than in rural areas. Note that the Todaro model of migration used in the text is therefore just a special case of having the employment probability lower in the urban area. As was pointed out in note 8, the equilibrium condition in the labor market is \( P = n \). The rural job probability (the coefficient of \( n \)) is unity. This is greater than the urban job probability \( P \), which is equal to the ratio of employment to the labor force in the Todaro model, and thus less than unity if there is any urban unemployment.

REFERENCES


Brent, R. J. (1991), "The Numbers Effect and the Social Discount Rate in Project Appraisal", *Fordham University, Department of Economics*.


