Exercises for Chapter 4 of Vinod’s “HANDS-ON INTERMEDIATE ECONOMETRICS USING R”

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Abstract

These are exercises to accompany the above-mentioned book with the URL: [http://www.worldscibooks.com/economics/6895.html](http://www.worldscibooks.com/economics/6895.html)

At this time all of the following exercises are suggested by H. D. Vinod (HDV) himself. Vinod invites readers to suggest new and challenging exercises (along with full and detailed answers) dealing with the discussion in Chapter 4 and related discussion from econometric literature by sending e-mail to vinod@fordham.edu. If the answers involve R programs, they must work. Readers may also suggest improvements to the answers and/or hints to existing exercises. If we include exercises and improvements suggested by readers, we promise to give credit to such readers by name. Furthermore, we will attach the initials of readers to individual exercises to identify the reader. Some R outputs are suppressed for brevity.

4 Exercises Mostly Based on Chapter 4 (Utility Theory) of the text

4.1 Exercise (Expected Utility Theory EUT)

1) How does EUT involve a probability distribution function?

2) Who wrote the first utility function around mid 19th century?

3) Who wrote the first joint utility function?
4) Verify that expected values for the utility associated with choices (a) and (b) on page 194 are the same.

5) Why concavity means risk aversion in Fig. 4.1 of the text [2] on page 195?

6) Explain why it makes a difference whether the expectation operator is used before or after the evaluation of the utility function U.

7) Show that the logarithmic utility satisfies Samuelson’s two axioms.

8) What is ‘elicitation’ of utility numbers from empirical choices among alternatives?

9) Define CARA and CRRA and evaluate it for the logarithmic utility.

10) List some types of observable human behavior inconsistent with the EUT.

11) Explain how the Taylor series links utility theory with the moments of the underlying probability distribution

HINTS on ANSWERS: 1) see p. 191 of the text [2]. 2) Gossen 3) Edgeworth 6) see eq (4.1.1). 7) See bottom page 196. 8) see mid page 197. 9) Use two derivatives of log(x) and see p. 211. 10) See p. 200. 11) See eq. (4.1.6) and (4.1.7).

4.2 Exercise (Non-Expected Utility Theory Non-EUT)

1) Describe the importance of unit square scaling of Lorenz curve in comparing income inequalities. Define the Gini coefficient for the Pareto distribution of incomes.

2) What are decision weights for Non-EUT?

3) Which analytical equation captures the four properties of utilities associated with Non-EUT?

4) Describe the axes of Fig. 4.3 and explain what it means to be above the 45 degree line. [Hint: cumulative probabilities as \( p_t = 1/T, 2/T, \ldots, 1 \) on the horizontal axis]
HINT: 1) Unit square simply means a square in the first quadrant formed by the two axes going from 0 to 1. It is equivalent to think of the axes going from 0 to 100 in percentages. Lorenz curves are drawn inside unit squares and allow us to compare totally different economies with reference to income inequality. The unit square method forces the scaling of data (income distribution) into comparable units for comparison of distributions and even for comparison of utilities. See pp 203-205. 2) See pp 206-207. 3) eq. 4.2.3

4.3 Exercise (Income Distributions)

Use the data from Table 1 of Vinod (J1985b, J of Business and Eco Stats, vol 3, pp 78-88) for 1979 for blacks and whites in US. Plot the two Lorenz curves in the same graph and compute the Gini coefficients for blacks and whites in 1979.

Does the difference between two Gini coefficients measure the economic distance between blacks and whites.

ANS:

```r
library(ineq)
# Lorenz Curve data for Blacks x=mid points of intervals#
x <- c(2.26, 5.34, 8.72, 12.49, 16.86, 25.33, 35.00, 147.00)
n <- c(25.7, 49.1, 65.8, 77.6, 86.3, 95.5, 99.5, 99.2, 100)
Lc1 <- Lc(x, n=n)
plot(Lc1,main="1979 Lorenz curve for blacks(solid line),
whites (dashed)")
# Lorenz Curve data for Whites x=mid points of intervals#
xw <- c(9.49, 17.10, 23.28, 29.51, 34.82, 50.00, 129.75, 187.00)
nw <- c(11.6, 27.3, 43.1, 57.4, 70.2, 86.6, 95.5, 100.00)
Lc2 <- Lc(xw, n=nw)
lines(Lc2, lty=2)
Gini(x)#Gini for blacks 0.6006324
Gini(xw)#Gini for whites 0.4915506
```

The difference between Gini coefficients merely measures inequality comparison within whites and within blacks separately. It says nothing about economic advantage of whites over blacks in US. To measure the latter one needs to use the “projected quantiles method” \((G^{-1}F(x_0))\) where both the cumulative density for blacks \(F\) and the cumulative density for whites \(G\) are needed, as shown in Vinod (J1885b).
4.3.1 Exercise (cumulative density)

Write an R function to compute the cumulative density when class intervals are of unequal length as in Vinod (J1885b).

ANSWER:

```r
# function to create cumulative frequencies from data # with unequal intervals etc
# if frequencies input were already cumulated, make cum=F
fn=function(x, limits, freq, cum=T) {
  cumf = freq # if frequencies input were already cumulated, make cum=F
  if (cum) cumf = cumsum(freq)
  ff = 0
  n = length(limits)
  if (x < limits[1]) ff = 0
  for (i in 1: (n-1)) {
    # print("i, n, limits[i], limits[i+1]"
    if (x > limits[i] && x <= limits[i+1]) ff = cumf[i]
  } # end of for loop
  if (x > limits[n]) {ff = 0}
  if (cum) ff = cumf[n]
  return(ff)
} # end of function fn
#
# example
# limits=c(0,5,10,15,20) # this is one longer
# freq=c(11.6, 15.7, 15.8, 14.3) # frequencies
# x=0:15
# y=rep(NA, length(x))
# for (i in 1:length(x)) {xx=x[i]; y[i]=fn(xx, limits, freq)}
# plot(x, y, typ="1")
library(MASS)
area(fn, freq=freq, limits=limits, a=0, b=15)
#[1] 215.4814 End of example
```
4.4 Exercise (Stochastic Dominance)

1) Explain the relation between four moments of the underlying probability distribution and four orders of stochastic dominance.

2) Explain how an investor can use this theory for choosing among mutual funds.

HINT: See Sec. 4.3 of the text [2].

4.5 Exercise (1SD Portfolio Comparisons)

Use seed 247. Use random series of length 60 each from the beta density used in the snippet #R4.3.1 (two sets of shape parameters) for first order stochastic dominance to create two artificial portfolios. Now use the ‘comp.portfo’ function of snippet #R4.3.6 on them. Note that if A is superior to B, ‘comp.portfo(A,B)’ provides four stochastic dominance measures (SD1 to SD4) that would be negative.

#Copy and paste all snippets of this chapter from the CD
set.seed(247)
x_a=rbeta(60, 2,5); mean(x_a) #0.3060741
x_b=rbeta(60, 3,6); mean(x_b) # 0.3065788
matplot(cbind(x_a,xb), typ="l",
main="Artificial portfolios 1SD of dashed returns")
library(fBasics)
basicStats(cbind(x_a,xb))
The above table shows that mean of xb is slightly larger, but we have no control over the variance, skewness and kurtosis.

```
layout(matrix(1:4, nrow = 2, ncol = 2))
dxa=density(xa)
n=length(dxa$x)
plot(dxa, main="Kernel smooth density for xa")
dxb=density(xb)
plot(dxb, main="Kernel smooth density for xb")
cdfxa=diffinv(dxa$y)
cdfxb=diffinv(dxb$y)
plot(dxa$x,cdfxa[2:(n+1)], typ="l",
    main="Cumulative density for xa",xlab="x", ylab="Cumulative density")
plot(dxb$x,cdfxb[2:(n+1)], typ="l",
    main="Cumulative density for xb",xlab="x", ylab="Cumulative density")
layout(matrix(1:1, nrow = 1, ncol = 1)) #back to normal plotting
```

The figures show that the two densities are close to each other and confirm what we see from descriptive statistics. Next we consider a set of reasonable choices of $\alpha$ and evaluate stochastic dominance properties.

Make sure that the typo discovered in March 2015 is incorporated in your R code. The last two arguments of stochdom should interchange. It should read:

```
stdo=stochdom(gp$wpa, gp$wpb, ibo$I.smallf, ibo$I.bigf)
out=matrix(NA,10,5)
alpp=c(seq(0.1, 0.9, by=0.1),0.99)
for (i in 1:10){
    out[i,1]=alpp[i]
    out[i,2:5]=comp.portfo(xa,xb, alp=alpp[i])
}
colnames(out)=c("alpha", "SD1", "SD2", "SD3", "SD4")
print(out)
```
Figure 1: Kernel smooth densities and cumulative densities for random data ‘xa’ and ‘xb’ from two beta densities
Figure 2: First order stochastically dominating portfolio (dashed line) with higher average return in artificial time series of returns.
Note that the portfolio ‘xb’ created from random numbers following the beta density is stochastically dominant over a similar portfolio ‘xa.’ The means of the two portfolios are 0.3060741 and 0.3065788 respectively, with a slightly higher value for ‘xb’. See Figure 4.4 on page 215 of the text [2]. The tabulated values are cumulated over all observations and the method involves numerical approximation to integrals of cumulative densities.

<table>
<thead>
<tr>
<th>alpha</th>
<th>SD1</th>
<th>SD2</th>
<th>SD3</th>
<th>SD4</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1,]</td>
<td>0.10</td>
<td>7.30788452</td>
<td>73.0763822</td>
<td>578.368982</td>
</tr>
<tr>
<td>[2,]</td>
<td>0.20</td>
<td>5.70326099</td>
<td>57.0050242</td>
<td>450.834537</td>
</tr>
<tr>
<td>[3,]</td>
<td>0.30</td>
<td>4.21454526</td>
<td>42.0949127</td>
<td>332.481290</td>
</tr>
<tr>
<td>[4,]</td>
<td>0.40</td>
<td>2.90026875</td>
<td>28.9296400</td>
<td>227.923225</td>
</tr>
<tr>
<td>[5,]</td>
<td>0.50</td>
<td>1.80998075</td>
<td>18.0024906</td>
<td>141.060483</td>
</tr>
<tr>
<td>[6,]</td>
<td>0.60</td>
<td>0.97629351</td>
<td>9.6376856</td>
<td>74.456514</td>
</tr>
<tr>
<td>[7,]</td>
<td>0.70</td>
<td>0.40845532</td>
<td>3.9264603</td>
<td>28.832322</td>
</tr>
<tr>
<td>[8,]</td>
<td>0.80</td>
<td>0.08959705</td>
<td>0.6995768</td>
<td>2.848903</td>
</tr>
<tr>
<td>[9,]</td>
<td>0.90</td>
<td>-0.02070820</td>
<td>-0.4471841</td>
<td>-6.691097</td>
</tr>
<tr>
<td>[10,]</td>
<td>0.99</td>
<td>0.01222014</td>
<td>-0.1614679</td>
<td>-4.861678</td>
</tr>
</tbody>
</table>

## Exercise (2SD Portfolio Comparisons)

Use seed 247. Use random series of length 60 each from the beta density similar to those used in the snippet #R4.3.1 (two sets of shape parameters) for second order stochastic dominance (2SD) based on the discussion below eq. (4.3.2) in the text [2] creating two artificial portfolios. Next use the ‘comp.portfo’ function of snippet #R4.3.6 on them. Comment on the link between expected utility theory, moments of density through the Taylor series (See eq 4.1.6 on page 201) in the context of numerical results.

```r
set.seed(247)
xc=rbeta(60, 3,6); mean(xc) #0.3535805
sd(xc) #0.1383265
xdd=rbeta(60, 2,4); mean(xdd)
xd=xdd+mean(xc)-mean(xdd)
mean(xd) #0.3535805
sd(xd) #0.1728417
## note xc and xd have same mean but xd has higher variance
```
## so xd is less desirable than xc

```r
matplot(cbind(xc,xd), typ="l",
    main="Artificial portfolios 2SD returns")
library(fBasics)
basicStats(cbind(xc,xd))
```

<table>
<thead>
<tr>
<th></th>
<th>xc</th>
<th>xd</th>
</tr>
</thead>
<tbody>
<tr>
<td>nobs</td>
<td>60.000000</td>
<td>60.000000</td>
</tr>
<tr>
<td>NAs</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.050071</td>
<td>0.084735</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.815099</td>
<td>0.812149</td>
</tr>
<tr>
<td>1. Quartile</td>
<td>0.262212</td>
<td>0.221339</td>
</tr>
<tr>
<td>3. Quartile</td>
<td>0.430922</td>
<td>0.446058</td>
</tr>
<tr>
<td>Mean</td>
<td>0.353580</td>
<td>0.353580</td>
</tr>
<tr>
<td>Median</td>
<td>0.325808</td>
<td>0.348666</td>
</tr>
<tr>
<td>Sum</td>
<td>21.214829</td>
<td>21.214829</td>
</tr>
<tr>
<td>SE Mean</td>
<td>0.017858</td>
<td>0.022314</td>
</tr>
<tr>
<td>LCL Mean</td>
<td>0.317847</td>
<td>0.308931</td>
</tr>
<tr>
<td>UCL Mean</td>
<td>0.399314</td>
<td>0.398230</td>
</tr>
<tr>
<td>Variance</td>
<td>0.019134</td>
<td>0.029874</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.138327</td>
<td>0.172842</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.631475</td>
<td>0.502989</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.692577</td>
<td>-0.273037</td>
</tr>
</tbody>
</table>

The above table shows that mean of xc is the same as mean of xd, the variance of xd is slightly larger, and we have no control over the skewness and kurtosis. Another way to compare the two data series is by kernel densities and cumulative densities as follows

```r
layout(matrix(1:4, nrow = 2, ncol = 2))
dxc=density(xc)
n=length(dxc$x)
plot(dxc, main="Kernel smooth density for xc")
dxd=density(xd)
plot(dxd, main="Kernel smooth density for xd")
cdfxc=diffinv(dxc$y)
cdfxd=diffinv(dxd$y)
plot(dxc$x,cdfxc[2:(n+1)], typ="l",
```

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The figures show that the two densities are close to each other and confirm what we see from descriptive statistics. Next we consider a set of reasonable choices of $\alpha$ and evaluate stochastic dominance properties.

```r
out=matrix(NA,10,5)
alpp=c(seq(0.1, 0.9, by=0.1),0.99)
for (i in 1:10){
    out[i,1]=alpp[i]
}
```

---

**Figure 3:** Kernel smooth densities and cumulative densities for random data ‘xc’ and ‘xd’ from two beta densities
Artificial portfolios 2SD returns

cbind(xc, xd)

Figure 4: Second order stochastically dominating portfolio with common mean but lower variance (solid line) of returns in artificial time series of returns.

\[\text{out}[i,2:5]=\text{comp.portfo}(xc, xd, \text{alp=alpp}[i])\]
\[\text{colnames(out)=c("alpha", "SD1", "SD2", "SD3", "SD4")}\]
\[\text{print(out)}\]

In these artificial data, in terms of order 1 stochastic dominance (1SD) both portfolios are identical. In terms of 2SD the variance of ‘xd’ being larger, it is dominated by ‘xc.’ We expect the SD2 column below to have negative values for realistic \(\alpha\) values of 0.5 to 0.8. The tabulated values are cumulated over all observations and the method involves numerical approximation to integrals of cumulative densities.
<table>
<thead>
<tr>
<th>alpha</th>
<th>SD1</th>
<th>SD2</th>
<th>SD3</th>
<th>SD4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>7.7440253</td>
<td>83.233141</td>
<td>707.32643</td>
<td>4925.8796</td>
</tr>
<tr>
<td>0.20</td>
<td>5.8712812</td>
<td>62.916211</td>
<td>533.17273</td>
<td>3704.5968</td>
</tr>
<tr>
<td>0.30</td>
<td>4.1428964</td>
<td>44.142873</td>
<td>372.14923</td>
<td>2574.8076</td>
</tr>
<tr>
<td>0.40</td>
<td>2.6307808</td>
<td>27.690506</td>
<td>230.87118</td>
<td>1582.6030</td>
</tr>
<tr>
<td>0.50</td>
<td>1.3958580</td>
<td>14.219155</td>
<td>114.95502</td>
<td>767.1110</td>
</tr>
<tr>
<td>0.60</td>
<td>0.4785509</td>
<td>4.167284</td>
<td>28.13094</td>
<td>154.3137</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.1091782</td>
<td>-2.334348</td>
<td>-28.48942</td>
<td>-248.0533</td>
</tr>
<tr>
<td>0.80</td>
<td>-0.3875477</td>
<td>-5.502497</td>
<td>-56.75099</td>
<td>-452.8521</td>
</tr>
<tr>
<td>0.90</td>
<td>-0.4063212</td>
<td>-5.872678</td>
<td>-61.20979</td>
<td>-491.8297</td>
</tr>
<tr>
<td>0.99</td>
<td>-0.2588691</td>
<td>-4.433834</td>
<td>-50.14727</td>
<td>-421.8879</td>
</tr>
</tbody>
</table>

### 4.7 Exercise (Small Cap Portfolio Comparisons)

Use ‘fPortfolio’ package small capitalization corporations’ funds data using ‘MARKET’ and ‘GG’ funds. Now compare them in terms of four orders of stochastic dominance. Plot the data and comment on the results. Include smooth kernel density plotting methods to show their densities and cumulative densities. Use the R function ‘diffinv’ for numerical integration.

```r
rm(list=ls()) #clean up old stuff
#Copy and paste all snippets of this chapter from the CD
library(fPortfolio)
smcap = as.timeSeries(data(SMALLCAP.RET)) #22 symbols
summary(smcap)
attach(smcap)
basicStats(cbind(MARKET,GG))
matplot(cbind(MARKET,GG), typ="l", main="Returns on two small cap portfolios")

layout(matrix(1:4, nrow = 2, ncol = 2))
dgg=density(GG)
n=length(dgg$x)
plot(dgg, main="Kernel smooth density for GG")
dmkt=density(MARKET)
plot(dmkt, main="Kernel smooth density for MARKET")
cdfgg=diffinv(dgg$y)
cdfmkt=diffinv(dmkt$y)
```
Figure 5: Two small capitalization portfolios called ‘MARKET’ (solid line) and ‘GG’ (dashed line) from package ‘fPortfolio’ and the function ‘diffinv’.
Figure 6: Kernel smooth densities and cumulative densities for small capitalization portfolios called ‘MARKET’ and ‘GG’.

plot(dgg$x, cdfgg[2:(n+1)], typ="l", 
     main="Cumulative density for GG", xlab="x", ylab="Cumulative density"
) plot(dmkt$x, cdfmkt[2:(n+1)], typ="l", 
     main="Cumulative density for MARKET", xlab="x", ylab="Cumulative density"
) layout(matrix(1:1, nrow = 1, ncol = 1)) #back to normal plotting

The R package called ‘cwhmisc’ [1] has more sophisticated cumulative density estimation from densities where a function called ‘smoothed.df’ carries out a (very quick and dirty) numerical integration, and then fits a spline to get a function which can be used to look up cumulative probabilities. This brings the fits on a comparable scale on the horizontal axis. Although interesting, it is not suitable for comparison of portfolios.
library(cwhmisc)
layout(matrix(1:4, nrow = 2, ncol = 2))
dgg=density(GG)
plot(dgg, main="Kernel smooth density for GG")
dmkt=density(MARKET)
plot(dmkt, main="Kernel smooth density for MARKET")
Fgg=smoothed.df(dgg)
Fmkt=smoothed.df(dmkt)
plot(Fgg, typ="l",main="Cumulative density for GG")
plot(Fmkt, typ="l",main="Cumulative density for MARKET")
layout(matrix(1:1, nrow = 1, ncol = 1)) #back to normal plotting

Note that the horizontal values in the figure above are approximate ones based on splines and do not exactly match those in the data.

<table>
<thead>
<tr>
<th></th>
<th>MARKET</th>
<th>GG</th>
</tr>
</thead>
<tbody>
<tr>
<td>nobs</td>
<td>60.000000</td>
<td>60.000000</td>
</tr>
<tr>
<td>NAs</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.156851</td>
<td>-0.454545</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.083346</td>
<td>1.000000</td>
</tr>
<tr>
<td>1. Quartile</td>
<td>-0.026746</td>
<td>-0.078333</td>
</tr>
<tr>
<td>3. Quartile</td>
<td>0.051776</td>
<td>0.069010</td>
</tr>
<tr>
<td>Mean</td>
<td>0.009145</td>
<td>0.021985</td>
</tr>
<tr>
<td>Median</td>
<td>0.018766</td>
<td>0.013423</td>
</tr>
<tr>
<td>Sum</td>
<td>0.548676</td>
<td>1.319092</td>
</tr>
<tr>
<td>SE Mean</td>
<td>0.006985</td>
<td>0.024519</td>
</tr>
<tr>
<td>LCL Mean</td>
<td>-0.004833</td>
<td>-0.027078</td>
</tr>
<tr>
<td>UCL Mean</td>
<td>0.023122</td>
<td>0.071047</td>
</tr>
<tr>
<td>Variance</td>
<td>0.002928</td>
<td>0.036071</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.054108</td>
<td>0.189924</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.682139</td>
<td>1.950717</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.060860</td>
<td>10.225168</td>
</tr>
</tbody>
</table>

Note that ‘GG’ has higher mean, variance, skewness and kurtosis. Higher mean and skewness are desirable. Higher variance and kurtosis are not desirable.

In the sequel we write a new R function to compute the stochastic dominance values for the range of $\alpha$ values used earlier.
Figure 7: Kernel smooth densities and cumulative densities for small capitalization portfolios called ‘MARKET’ and ‘GG’ based on package ‘cwhmisc’ to get comparable but approximate cumulative densities.
compall=function(xa,xb){  #New function for all comparisons
out=matrix(NA,10,5)
alpp=c(seq(0.1, 0.9, by=0.1),0.99)
for (i in 1:10){
  out[i,1]=alpp[i]
  out[i,2:5]=comp.portfo(xa,xb, alp=alpp[i])
}
colnames(out)=c("alpha", "SD1", "SD2", "SD3", "SD4")
#print(out)
return(out)} #end of the new function
compall(MARKET,GG)

Note that negative values in the function ‘comp.portfo(xa,xb)’ of SD criteria suggest dominance of ‘xa’ over ‘xb’. Since ‘xa’ is MARKET, ‘GG’ is inferior to ‘MARKET’ under all SD1 to SD4 criteria for all \( \alpha \) values.

<table>
<thead>
<tr>
<th>alpha</th>
<th>SD1</th>
<th>SD2</th>
<th>SD3</th>
<th>SD4</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1,]</td>
<td>0.10</td>
<td>-16.135552</td>
<td>-286.74540</td>
<td>-3883.7781</td>
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<td>-14.054871</td>
<td>-251.13215</td>
<td>-3411.0124</td>
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<td>[3,]</td>
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<td>-10.079495</td>
<td>-183.25745</td>
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<tr>
<td>[5,]</td>
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<tr>
<td>[6,]</td>
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<td>-123.00723</td>
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<tr>
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<tr>
<td>[9,]</td>
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<tr>
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<td>0.99</td>
<td>-1.391487</td>
<td>-36.41349</td>
<td>-582.0341</td>
</tr>
</tbody>
</table>

The generally negative values throughout above table suggest that investment in ‘GG’ is generally inferior than in ‘MARKET’ portfolio for all stochastic dominance measures, whether or not the sample moments show similar superiority. This result seems to hold less strongly (smaller negative numbers for large alpha) for EUT-compliant agents (having alpha close to 1) than for non-compliant agents.

Now we comment on the link between expected utility theory (EUT) and moments of density through the Taylor series (See eq 4.1.6 on page 201) in the context of numerical results. The mean, variance, skewness and kurtosis are related to the first four moments of the density. However they are summary measures and higher moments are not easily reliably estimated, especially when the underlying distributions have more than one mode as seen from...
our smooth Kernel plots. The methods used in ‘comp.portfo’ are based on empirical CDF of actual data and involve a more detailed approach to the density functions implied by the data. The dominance measures computed from these densities are more accurate, except that when we use numerical integration (by matrix multiplication) we might introduce some errors.

4.8 Exercise (Further Small Cap Portfolio Comparisons)
Use the SMALLCAP.RET data for 21 stocks and the MARKET from the package fPortfolio. Use it instead of the artificial data in snippet 4.3.6 and indicate which two stock symbols you will buy (highest returns) and which two symbols you will sell (lowest rank in returns) in terms of 4 orders of stochastic dominance and choose two values of alpha at 0.9 and 0.5. Always compare each stock with the MARKET (one of the columns in that data set).

4.9 Exercise (SPIDER Sector Portfolio Comparisons)
Use the SPISECTOR.RET data for 10 sectors from the package fPortfolio. Use SPI the first sector as the reference sector. Use these data instead of the artificial data in snippet 4.3.6 indicate which two sector symbols you will buy (highest returns) and which two symbols you will sell (lowest rank in returns) in terms of 4 orders of stochastic dominance and choose two values of alpha at 0.5 and 0.2.

4.10 Exercise (Beta Density Comparisons)
Use 500 randomly generated returns from the beta density consistent with the two examples plotted in Figures 4.4 and 4.5 and apply the software to compute whether the correct data dominates with reference to first and second order stochastic dominance.

4.11 Exercise (R Software Understanding)
Discuss the inputs, outputs and the purpose of last three functions in the snippets in Chapter 4 of your text [2]. Explain why negative values in the function ‘comp.portfo(xa,xb)’ of SD criteria suggest dominance of ‘xa’ over
‘xb’. How would you rewrite the functions so that positive values imply superior portfolio. (Hint: see line 6 on textbook snippet R4.3.5 where we have (wpa-wpb) the difference of weighted probabilities. You may want to replace it with (wpb-wpa) in the snippet so that positive values of SD1 to SD4 will suggest ‘xa’ to be superior to ‘xb’).

References

